Impact of Education Subsidies and Taxation on Wealth and Human Capital Accumulation

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Abstract
This paper proposes an economic growth model of endogenous wealth and human capital with government's subsidy policies on education. The economic system is built on the basis of the Solow growth model and the Uzawa-Lucas two-sector model. We take into account three ways of accumulating human capital: learning by producing, learning by education, and learning by consuming. The model describes a dynamic interdependence among wealth accumulation, human capital accumulation, division of labor, and government taxation. We simulate the model to demonstrate the existence of equilibrium points and motion of the dynamic system. We also examine the effects of changes in the propensity to receive education, efficiency of learning, and efficiency of education upon dynamic paths of the system.

Keywords: economic growth; learning by consuming; learning by education; government subsidy; propensity to receive education

JEL classification: O41, H2, I26

Introduction

In contemporary economics, human capital is generally considered as a key determinant of economic growth (Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; and Castelló-Climent and Hidalgo-Cabrillana, 2012). Dynamic interdependence between economic growth and human capital is currently a main topic in economic theory. As education is an important way of accumulating human capital, many models have been proposed to examine interdependence between education and economic growth. As far as formal modeling of education and economic growth is concerned, the work by Lucas (1988) has caused a great interest in the issue among economists. In fact, the first formal dynamic growth model with education was proposed by Uzawa (1965). The Uzawa-Lucas model has

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been extended and generalized in various directions. One of these directions is to examine the effects of taxation on economic growth, education, and human capital accumulation (e.g., Jones et al. 1993; Stokey and Rebelo, 1995; Mino, 1996, 2001; Zhang, 2003; Alonso-Carrera and Freire-Seren, 2004; De Hek, 2005; Chakraborty and Gupta, 2009; and Sano and Tomoda, 2010). The main purpose of our study is to introduce government subsidy on education into the Uzawa-Lucas two sector model. Education is nowadays often provided by the public or subsidized by the state. According to Arcalean and Schiopu (2010), "in the OECD countries significant cofinancing takes place at the tertiary and pre-primary education levels, where 24 % and respectively 19% of total funds come from private sources. A similar pattern is visible in developing countries, particularly in higher education, where participation increased steadily during the last two decades." Adam Smith recognized the importance of state intervention education due to market failures. A hotly discussed topic in higher education is about the declination of public support for education and above CPI increases in tuition (e.g., Fethke, 2011).

There are many empirical studies about interactions among growth, human capital accumulation, and education subsidies. In the last decades higher education enrollment has increased in many countries. Although the trend is quite common among industries economies, there are great differences in the provision of higher education across countries. For instance, countries like Denmark and Canada fully rely on public provision, other countries like Japan and Korea are mostly supported by private individuals for higher education (Bergh and Fink, 2009). Bergh and Fink (2009) observe two patterns in comparing the provision of higher education and labor market outcomes across countries. The first is that a large share of private providers tends to be associated with higher returns to education. The second pattern is that there does not seem to be a systematic relation between the structure of higher education and the overall degree income inequality. Caucutt and Kumar (2003) construct a dynamic general equilibrium model to study the effects of higher education subsidies in the US. They examine different combinations of taxation and subsidy policies. Some studies examine the implications of physical resources devoted to education and its role in economic growth (e.g., Rangazas, 2000, 2002; and Chanda, 2008). As mentioned by Chanda (2008), over the last three decades returns to higher education have increased while the household savings rate has fallen to almost zero in the US. Chanda builds a representative agent model where savings fall as an outcome of an exogenously driven increase in the return to education. The model shows that a part of the decline in savings may reflect a relative reallocation of the resources from wealth accumulation to human capital accumulation. As shown by Su (2006), the allocation of the budget is different across countries. In general, in less developed economies public budget allocation is characterized by exclusive participation and large schooling expenditure at higher education at the expense of basic education; in developed economies, the budget allocation tends to be more balanced. This study also tries to discuss some of the empirical findings in the literature.

We analyze the link between government education spending and growth by constructing a two-sector endogenous growth model. Although issues related to interdependence between growth, education and government's subsidy on education have been examined from different perspectives, this paper makes a unique approach to the issue in that it models household's decision on education with an alternative approach proposed by Zhang (1993), and considers the sources of human capital via three ways: Arrow's learning by doing, Uzawa's learning by education, and Zhang's creative leisure within a general equilibrium framework. Since the model is characterized of general equilibrium among various forces of households' decisions, economic structures, and human capital and wealth changes, in comparison with most of the theoretical models with partial analyses to the issues our model can properly deal with the impact of government's spending on education in a comprehensive perspective. We build a growth model with physical capital and human capital accumulation. Like the Uzawa-Lucas mode, the economy consists of industrial and education sectors. We combine the economic mechanisms in the three key growth models - Solow's growth model, Arrow's learning by doing model, the Uzawa-Lucas education model into a single comprehensive framework. The paper is an extension of Zhang's previous works (2007, 2014) by taking account of government's intervention in education. The synthesis of these growth models within a single framework is analytically tractable because we propose an alternative approach to consumers' behavior. The paper is organized as follows. Section 2 introduces the basic model with wealth accumulation and human capital accumulation with government subsidy on education. Section 3 simulates the model. Section 4 carries out comparative dynamic analysis with regard to some parameters. Section 5 concludes the study.

The Basic Model

The economy has one production sector and one education sector. Most aspects of the production sector are similar to the standard one-sector growth model (Burmeister and Dobell, 1970; Azariadis, 1993; and Barro and Sala-i-Martin, 1995). It is assumed that there is only one (durable) good in the economy under consideration. Households own assets of the economy and distribute their incomes to consumption, education and wealth accumulation. The production sectors or firms use physical capital and labor as inputs. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production. We assume a homogenous and fixed population N_0. Let $T(t)$ and $T_e(t)$ represent, respectively, the work time and study time of a representative household. The total work time, $N(t)$, is $T(t)N_0$.

The labor force is distributed between the two sectors. We select the commodity to serve as numeraire, with all the other prices being measured relative to its price. We assume that wage rates are identical among all professions. The total capital stock of physical capital, $K(t)$, is allocated between the two sectors. We use $N_e(t)$ and $K_e(t)$ to stand for the labor force and capital stocks employed by the education sector, and $N_i(t)$ and $K_i(t)$ for the labor force and capital stocks employed by the production sector. As labor and capital are assumed fully employed, we have:

$$K_i(t) + K_e(t) = K(t), \quad N_i(t) + N_e(t) = N(t).$$

We rewrite the above relations as follows:

$$n_i(t)k_i(t) + n_e(t)k_e(t) = k(t), \; n_i(t) + n_e(t) = 1, \tag{1}$$

in which

$$k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad n_j(t) \equiv \frac{N_j(t)}{N(t)}, \quad k(t) \equiv \frac{K(t)}{N(t)}, \quad j = i, e.$$

The production sector

We use $H(t)$ to present for the level of human capital of the population. We assume that production is to combine 'qualified labor force', $H^m(t)N_i(t)$, and physical capital, $K_i(t)$, where the parameter, m, describes the efficiency of how effectively the population uses human capital. We use the conventional production function to describe a relationship between inputs and output. We use $F_i(t)$ to present the output level of the production sector at time t. The production function is specified as follows:

$$F_i(t) = A_i\, K_i^{\alpha_i}(t)\big(H^m(t)N_i(t)\big)^{\beta_i}, \quad A_i,\, \alpha_i,\, \beta_i > 0, \quad \alpha_i + \beta_i = 1.$$

where A_i, α_i, and β_i are positive parameters. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest, $r(t)$ and wage rate, $w(t)$ are determined by markets. Hence, for any individual firm $r(t)$ and $w(t)$ are given at each point of time. The marginal conditions are given by:

$$r(t) + \delta_k = \frac{\alpha_i\, \bar{\tau}_i\, F_i(t)}{K_i(t)} = \bar{\tau}_i\, \alpha_i\, A_i\, H^{m\beta_i}\, k_i^{-\beta_i},$$
$$w(t) = \frac{\beta_i\, \bar{\tau}_i\, F_i(t)}{N_i(t)} = \bar{\tau}_i\, \beta_i\, A_i H^{m\beta_i}\, k_i^{\alpha_i}, \qquad \textbf{(2)}$$

where δ_k is depreciation rate of physical capital and $\bar{\tau}_i \equiv 1 - \tau_i$, τ_i being the fixed tax rate on the industrial output.

Accumulation of human capital and the education sector

We assume that there are three sources of improving human capital, through education, "learning by producing", and "learning by leisure". Arrow (1962) first introduced learning by doing into growth theory; Uzawa (1965) took account of trade-offs between investment in education and capital accumulation, and Zhang (2007, 2014) introduced impact of consumption on human capital accumulation (via the so-called creative leisure) into growth theory. Synthesizing the three resources of

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learning with a single learning equation, we propose that human capital dynamics is given by:

$$\dot{H} = \frac{\upsilon_e F_e^{a_e}\left(H^m T_e N_0\right)^{b_e}}{H^{\pi_e} N_0} + \frac{\upsilon_i F_i^{a_i}}{H^{\pi_i} N_0} + \frac{\upsilon_h C^{a_h}}{H^{\pi_h} N_0} - \delta_h H, \qquad \textbf{(3)}$$

where δ_k (>0) is the depreciation rate of human capital, υ_e, υ_i, υ_h, a_e, b_e, a_i, , and a_h are non-negative parameters. The signs of the parameters, π_e, π_i,, and π_h are not specified as they can be either negative or positive.

The term, $\upsilon_e F_e^{a_e}\left(H^m T_e N_0\right)^{b_e} / H^{\pi_e} N_0$, describes the contribution to human capital improvement through education. Human capital tends to increase with an increase in the level of education service, F_e, and in the (qualified) total study time, $H^m T_e N_0$. We take account of learning by producing effects in human capital accumulation by the term $\upsilon_i F_i^{a_i} / H^{\pi_i}$. This term implies that contribution of the production sector to human capital improvement is positively related to its production scale and is dependent on the level of human capital. We take account of learning by consuming by the term, $\upsilon_h C^{a_h} / H^{\pi_h} N_0$. This term can be interpreted similarly as the term for learning by producing. Zhang (2007) first introduced this equation to growth theory. In contemporary economies, human capital is evidently closely related to leisure activities such as different club activities, travelling different parts of the world, playing computer games, watching TV, and doing sports, It should be noted that this paper differs from Zhang in that this paper treats education time as an endogenous variable and the education sector as a as private sector, while Zhang treats the education sector as a public sector and education is free for individuals. In another study by Zhang (2007), education is a private sector and fully financially by individuals. This study differs from Zhang in that education is subsided by the government through taxing the households as well as the production and education sectors. The taxing structure is more realistic, even though we neglect issues related to how tax rates may be dependent on different factors, such as the stage of economic development and income distribution, of the economic system. The model in this study is a generalization of Zhang's two models. In the literature of education and economic growth, there are some growth models with public or/and private education. Here, we refer to only a few early studies on the issues. Eckstein and

Zilcha (1994) propose an overlapping model with compulsory schooling. In Kaganovich and Zilcha (1999) the government allocates tax revenues towards public investment in education and social security benefits. Epple and Romano (1998) and Caucutt (2002) examine school choice with peer effects. In Cardak (2004) and Chen (2005), mixed education systems are presented.

We assume that the education sector is also characterized of perfect competition. Here, we neglect any government's financial support to the education sector. It can be seen that it is not difficult to introduce government's support to the education sector by changing, for instance, the tax rate on the education sector with the subsidy rate. Here, we assume that the education sector charges students $p(t)$ per unit of time. The education sector pays teachers and capital with the market rates. The cost of the education sector is given by $w(t)N_e(t)+r(t)K_e(t)$. The total education service is measured by the total education time received by the population, T_eN_0. The production function of the education sector is assumed to be a function of $K_e(t)$ and $N_e(t)$. We specify the production function of the education sector as follows:

$$F_e(t) = A_e K_e^{\alpha_e} \left(H^m N_e\right)^{\beta_e}, \ \alpha_e, \beta_e > 0, \ \alpha_e + \beta_e = 1, \tag{4}$$

where A_e, α_e and β_e are positive parameters. The education sector maximizes the following profit:

$$\pi(t) = \bar{\tau}_e \, p(t) A_e \, K_e^{\alpha_e} \left(H^m \, N_e\right)^{\beta_e} - \left(r(t) + \delta_k\right) K_e(t) - w(t) N_e(t),$$

where $\bar{\tau}_e \equiv 1 - \tau_e$, and τ_e is the fixed tax rate on the education service.

For given $p(t)$, $H(t)$, $r(t)$, and $w(t)$, the education sector chooses $K_e(t)$ and $N_e(t)$ to maximize the profit. The optimal solution is given by

$$r + \delta_k = \alpha_e \, \bar{\tau}_e \, A_e \, p \, H^{m\beta_e} \, k_e^{-\beta_e}, \ \ w = \beta_e \, \bar{\tau}_e \, A_e \, p \, H^{m\beta_e} \, k_e^{\alpha_e} . \tag{5}$$

The demand for labor force for given price of education, wage rate and level of human capital is given by

$$N_e = K_e \left(\frac{\beta_e \bar{\tau}_e A_e p H^{m\beta_e}}{w} \right)^{1/\alpha_e}.$$

We see that the demand for teachers increases in the total productivity of the education sector, the price and level of human capital, and decreases in the wage rate and the tax rate on the education sector.

Consumer behaviors

Consumers decide the time of education, the consumption level of commodities, and amount of saving. Different from the optimal growth theory in which utility defined over future consumption streams is used, we use an alternative approach to household proposed by Zhang (1993). To describe he behavior of consumers, we denote per capita wealth by $\bar{k}(t)$, where $\bar{k}(t) \equiv K(t)/N_0$. By the definitions, we have $\bar{k}(t) = T(t)k(t)$. We use τ_h, τ_w and τ_c to stand for the fixed tax rates on the wealth income, wage income and consumption. Per capita current income from the interest payment $\bar{\tau}_h r(t)\bar{k}(t)$ and the wage payment $\bar{\tau}_w T(t)w(t)$ is given by:

$$y(t) = \bar{\tau}_h r(t)\bar{k}(t) + \bar{\tau}_w T(t)w(t),$$

where $\bar{\tau}_h \equiv 1 - \tau_h$ and $\bar{\tau}_w \equiv 1 - \tau_w$. We call *y(t)* the current income in the sense that it comes from consumers' payment for "qualified" efforts and consumers' current earnings from ownership of wealth. The total value of wealth that consumers can sell to purchase goods and to save is equal to $\bar{k}(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by:

$$\hat{y}(t) = y(t) + \bar{k}(t) = (1 + \bar{\tau}_h r(t))\bar{k}(t) + \bar{\tau}_w T(t)w(t). \tag{6}$$

The disposable income is used for saving, consumption, and education. Let *τ(t)* stand for the subsidy per unit of time people receive from the government for education. In the literature of growth with education subsidies, there are different ways of modeling household behavior and budget constraints (e.g., Zhang, 2003; Blankenau and Simpson, 2004; Bovenberg and Jacobs, 2005; Chakraborty and Gupta, 2009; Booth and Coles, 2010). Hence, the education cost is the price minus the subsidy rate, $p(t)$-$\tau(t)$. At each point of time, a consumer would distribute the total available budget among saving, $s(t)$, consumption of goods, $c(t)$ and education, $T_e(t)$. The budget constraint is given by:

$$\bar{\tau}_c\, c(t) + s(t) + \left(p(t) - \tau(t)\right)T_e(t) = \left(1 + \bar{\tau}_h\, r(t)\right)\bar{k}(t) + \bar{\tau}_w\, T(t) w(t), \quad \textbf{(7)}$$

where $\bar{\tau}_c \equiv 1 + \tau_c$. The consumer is faced with the following time constraint:

$$T(t) + T_e(t) = T_0,$$

where T_0 is the total available time for work and study. Substituting this function into the budget constraint (7) yields

$$\bar{\tau}_c\, c(t) + s(t) + \bar{p}(t)T_e(t) = \bar{y}(t) \equiv \left(1 + \bar{\tau}_h\, r(t)\right)\bar{k}(t) + \bar{\tau}_w\, T_0\, w(t), \quad \textbf{(8)}$$

where $\bar{p}(t) \equiv p(t) + w(t) - \tau(t)$. The right-hand side is the "potential" income that the consumer can obtain. The left-hand side is the total cost of consumption and saving plus opportunity cost of education (i.e., wT_e). At each point of time, consumers have three variables, the level of consumption, the level of saving, and the education time, to decide. We assume that consumers' utility function is a function of $c(t)$, $s(t)$, and $T_e(t)$ as follows:

$$U(t) = c^{\xi_0}(t) s^{\lambda_0}(t) T_e^{\eta_0}(t), \quad \textbf{(9)}$$

where ξ_0 is called the propensity to consume, λ_0 the propensity to own wealth, and η_0 the propensity to obtain education. For the representative consumer, $w(t)$ and $r(t)$ are given in markets. Maximizing $U(t)$ subject to (8) yields

$$c(t) = \xi\, \bar{y}(t), \;\; s(t) = \lambda\, \bar{y}(t), \;\; \bar{p}\, T_e(t) = \eta\, \bar{y}, \quad \textbf{(10)}$$

where

$$\xi \equiv \frac{\rho \xi_0}{\bar{\tau}_c}, \ \lambda \equiv \rho \lambda_0, \ \eta \equiv \rho \eta_0, \ \rho = \frac{1}{\xi_0 + \lambda_0 + \eta_0}.$$

The demand for education is given by $T_e = \eta \bar{y} / \bar{p}$. The demand for education falls in the price of education and rises in the wealth income and subsidy rate. A rise in the propensity to receive education increases the education time when the other variables are fixed.

We now find dynamics of capital accumulation. According to the definition of *s(t),* the change in the household's wealth is given by:

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) = \lambda \bar{y}(t) - \bar{k}(t). \quad \textbf{(11)}$$

For the education sector, the demand and supply balances at any point of time:

$$T_e N_0 = F_e(t). \quad \textbf{(12)}$$

As the government's tax is spent only on subsidizing education, we have:

$$\tau(t) T_e(t) = \frac{\tau_i F_i(t)}{N_0} + \frac{\tau_e \, p F_e(t)}{N_0} + \tau_c \, c(t) + \tau_h \, r(t) \bar{k}(t) + \tau_w \, T(t) w(t). \quad \textbf{(13)}$$

As output of the production sector is equal to the sum of the level of consumption, the depreciation of capital stock and the net savings, we have:

$$C(t) + S(t) - K(t) + \delta_k K(t) = F_i(t),$$

where *c(t)* is the total consumption, $S(t) - K(t) + \delta_k K(t)$ is the sum of the net saving and depreciation, that is

$$C(t) = c(t) N_0, \ S(t) = s(t) N_0.$$

We have thus built the dynamic model. We now examine dynamics of the model.

The Dynamics and Its Properties

This section examines dynamics of the model.

First, we show that the dynamics can be expressed by the two-dimensional differential equations system with $k_i(t)$ and $H(t)$ as the variables.

Lemma

The dynamics of the economic system is governed by the 2-dimensional differential equations

$$\dot{k}_i(t) = \tilde{\Omega}_i(k_i, H),$$
$$\dot{H}(t) = \tilde{\Omega}_h(k_i, H), \tag{14}$$

where the functions $\tilde{\Omega}_i(k_i, H)$ and $\tilde{\Omega}_h(k_i, H)$ are functions of $k_i(t)$ and $H(t)$ defined in the Appendix. Moreover, all the other variables can be determined as functions of $k_i(t)$ and $H(t)$ at any point of time by the following procedure: $k(t)$ by (A9) $\rightarrow$ $T(t)$ by (A8) $\rightarrow$ $T_e(t) = T_0 - T(t)$ $\rightarrow$ $\bar{k}(t) = k(t)T(t)$ $\rightarrow$ $k_h(t) = \alpha k_0(t)$ $\rightarrow$ $\bar{y}(t)$ by (A10) $\rightarrow$ $p(t)$ by (A2) $\rightarrow$ $n_i(t)$ and $n_h(t)$ by (A3) $\rightarrow$ $r(t)$ and $w(t)$ by (2) $\rightarrow$ $c(t)$ and $s(t)$ by (10) $\rightarrow$ $N(t) = N_0 T(t)$ $\rightarrow$ $N_j(t) = n_j(t)N(t), j = i, e$ $\rightarrow$ $K(t) = k(t)N(t)$ $\rightarrow$ $K_j(t) = k_j(t)N_j(t)$ $\rightarrow$ $F_j(K_j(t), N_j(t))$ $\rightarrow$ $\tau(t)$ and $s(t)$ by (13).

The differential equations system (14) contains two variables, $k_i(t)$ and $H(t)$. Although we can analyze its dynamic properties as we have explicitly expressed the dynamics, we omit analyzing the model as the expressions are too complicated. Instead, we simulate the model to illustrate behavior of the system. In the remainder of this study, we specify the depreciation rates by $\delta_k = 0.05$, $\delta_h = 0.04$, and let $T_0 = 1$. The requirement $T_0 = 1$ will not affect our analysis. We specify the other parameters as follows

$$\alpha_i = 0.34,\ \alpha_e = 0.55,\ \lambda_0 = 0.6,\ \xi_0 = 0.08,\ \eta_0 = 0.007,$$
$$N_0 = 50000,\ A_i = 0.9,\ A_e = 0.7,\ m = 0.7,\ v_e = 0.8,$$
$$v_i = 2.5,\ v_h = 0.7,\ a_e = 0.3,\ b_e = 0.5,\ a_i = 0.4,\ a_h = 0.1,$$
$$b_h = 0.3,\ \pi_e = -0.1,\ \pi_i = 0.7,\ \pi_h = 0.1,$$
$$\tau_i = \tau_w = \tau_h = \tau_e = \tau_c = 0.005. \quad \textbf{(15)}$$

The propensity to save is 0.6 and the propensity to consume education is 0.007. The propensity to consume goods is 0.08. The technological parameters of the two sectors are specified at $A_i = A_e = 0.9$. The conditions $\pi_e=-0.1$, $\pi_i=0.7$, and $\pi_e=0.1$, mean respectively that the learning by education exhibits increasing effects in human capital; the learning by producing exhibits (strong) decreasing effects in human capital; and the learning by consuming exhibits (weak) increasing effects in human capital. By (14), an equilibrium point of the dynamic system is given by

$$\Omega(k_i, H) = 0,$$
$$\tilde{\Omega}(k_i, H) = 0. \quad \textbf{(16)}$$

Simulation demonstrates that the above equations have the following unique equilibrium solution

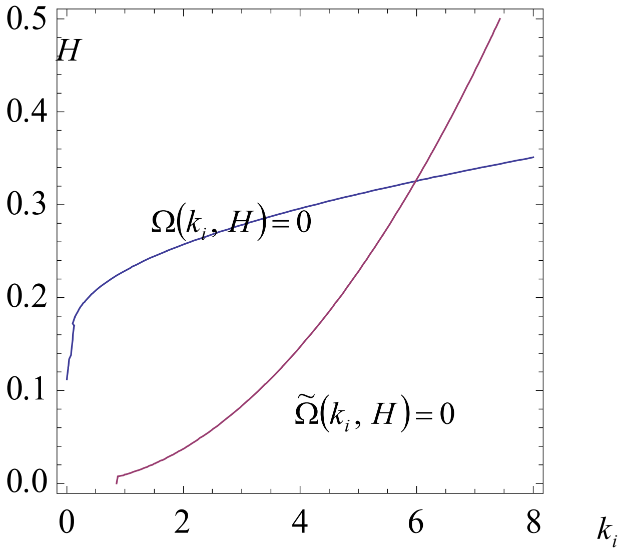
$$k_i = 5.98,\ H = 0.33.$$

We plot the solution of (16) as in Figure 1. Further simulation shows that (16) has a unique equilibrium.

The equilibrium values of the other variables are given by the procedure in Lemma 1. We list them as follows:

$$H = 0.33,\ N = 47047.8,\ N_i = 45651.5,\ N_e = 1396.26,$$
$$K_i = 273171,\ K_e = 19822.7,\ f_i = 0.98,\ f_e = 1.32,$$
$$F_i = 44931.7,\ F_e = 1835.6,\ k = 6.23,\ k_i = 5.98,\ k_e = 14.20,$$
$$p = 0.68,\ w = 0.65,\ \tau = 0.20,\ \bar{k} = 5.86,\ T = 0.94,$$
$$T_e = 0.06,\ c = 0.78. \quad \textbf{(17)}$$

Fig. 1. Solution of formula (16)



The consumer spends about 6 percent of the total available time for study. The subsidy per unit of education time is 0.2, while the education fee is 0.68. Hence, the government subsidizes about 30 percent of the education cost. It is straightforward to calculate the two eigenvalues as follows: -0.121 and -0.04. As the two eigenvalues are negative, the unique equilibrium is locally stable. Hence, the system always approaches its equilibrium if it is not far from the equilibrium. We are now concerned with motion of the system. We specify initial conditions as follows: $k_i(0)=6.2$, and $H(0)=0.3$. The simulation result is plotted in Figure 2. The level of the human capital increases from the initial state to the equilibrium value. The wealth and consumption experience a kind of *J* - curve process. It first experiences declination in per capita levels of consumption and wealth. After some time these variables start to increase. The education time slightly declines and soon begins to increase. During the simulation period, the education fee and subsidizing fee increase.

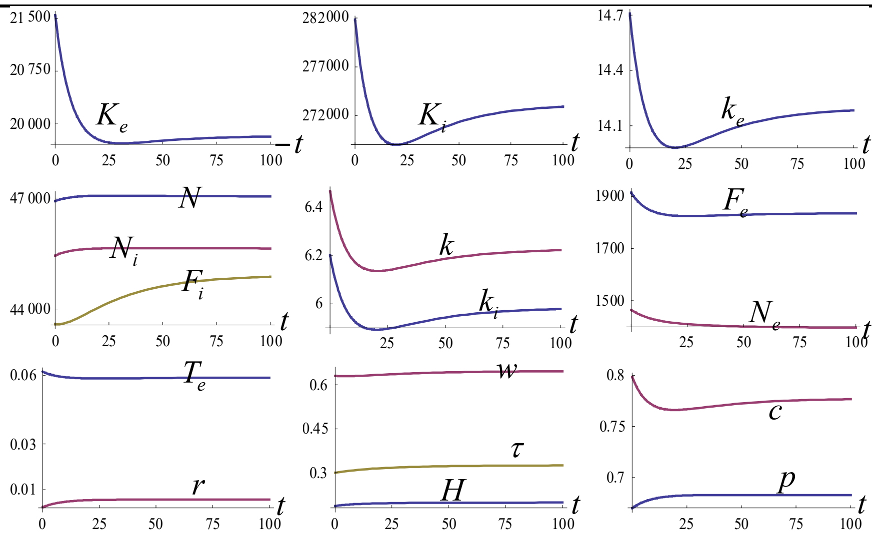


Fig. 2.
The motion of the economic system

Comparative Dynamic Analysis

We now examine impact of changes in different exogenous factors on the dynamic processes of the system. We introduce variable $\overline{\Delta} x(t)$ to stand for the change rate of the variable, *x(t)*, in percentage due to changes in the parameter value.

A rise in the total productivity of the education sector

First, we examine the case that all the parameters, except the total productivity of the education sector, A_e, are the same as in (17). We increase as follows: $A_e : 0.7 \Rightarrow 0.9$. The simulation results are plotted in Figure 3. In order to examine how each variable is affected over time, we should follow the motion of the entire system as each variable is related to the others in the dynamic system. When A_e is increased, according to the demand and supply equation, $T_e N_0 = F_e$ the education time will be increased initially. As T_e is increased, the education fee will be increased. As more time is devoted to education, human capital rises, which also results in increase in the wage rate. As the household initially devotes more income to education, the levels of per capita consumption and wealth fall. But as income is increased, the consumption and wealth soon begin to increase. As the total productivity is increased, the education sector uses less capital and labor. The work time falls slightly.

Although the total tax income from wage, wealth and outputs of the two sectors are increased, the subsidy rate falls as the total education time is increased dramatically. Hence, the government subsidizes students less per unit of study time.

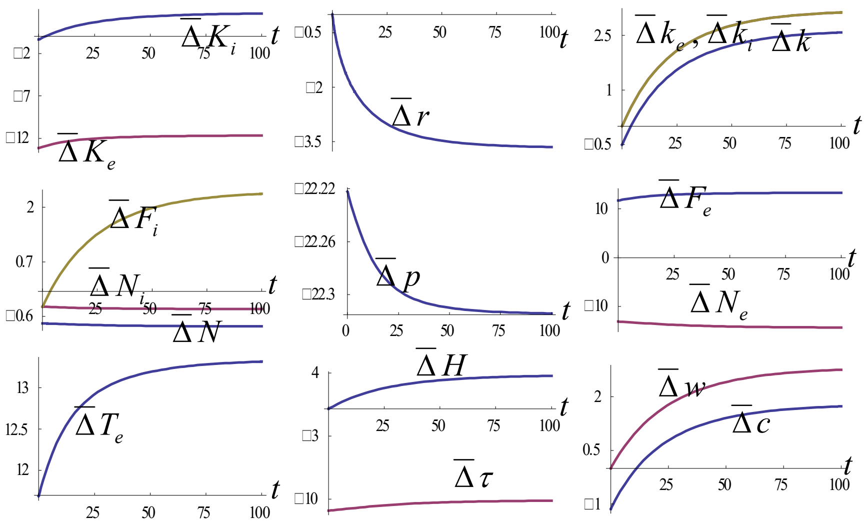


Fig. 3.
A rise in the total productivity of the education sector

A rise in the tax rate on the wage income

In Trostel (1993), a proportional income tax significantly reduces investments in education. This occurs because individuals' cost of education is not tax-deductible in the model. Jacobs (2005) studies optimal income taxation by taking account human capital accumulation as a dimension of labor supply with differences among households in the ability to learn. Taxation affects utilization of human capital through labor supply responses. Moreover, according to Jacobs' analysis, the costs of education that are not deductible from the income tax distort the learning decision. We now examine effects of change in tax rates on education and economic variables. We increase the tax rate on the wage income in order to further support education in the following way: $\tau_w : 0.005 \Rightarrow 0.015$. The simulation results are plotted in Figure 4. As the tax income is increased, more public money is spent on education initially. The subsidy rate is increased. As the education is increased only slightly and the subsidy rate is increased much, it costs less for students to study. Hence, demand for education will be increased. As the

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population spends more time on education and the output level and consumption are reduced but slightly initially, human capital is improved. The rate of interest is increased. The wealth and capital intensities initially rise, and then fall, and rise in the long term. It can be seen that our prediction is different from Trostel (1993). The reason is that in our model individuals' cost of education fall as a consequence of raising income tax. We also simulate the impact of τ_c increases from 0.005 to 0.015. As far as the change directions in the variables are concerned, the effects are similar to the effects of the change in τ_w.

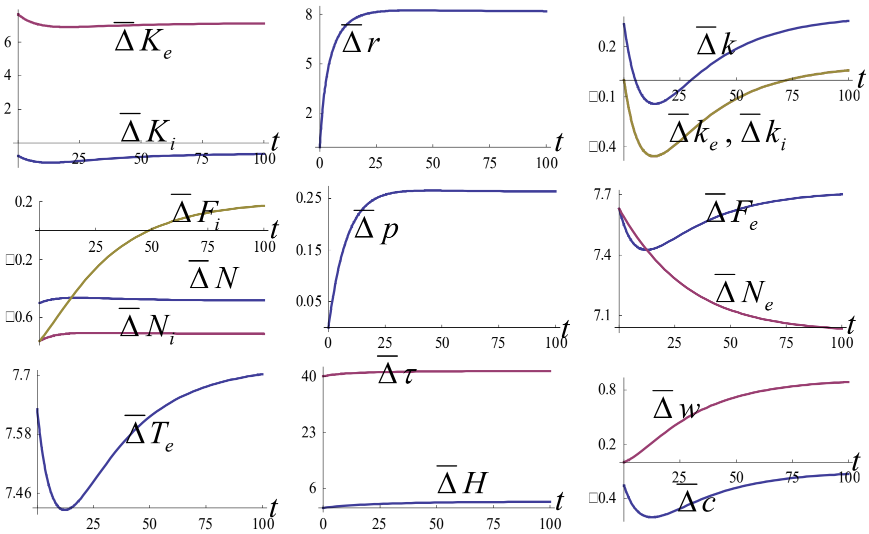
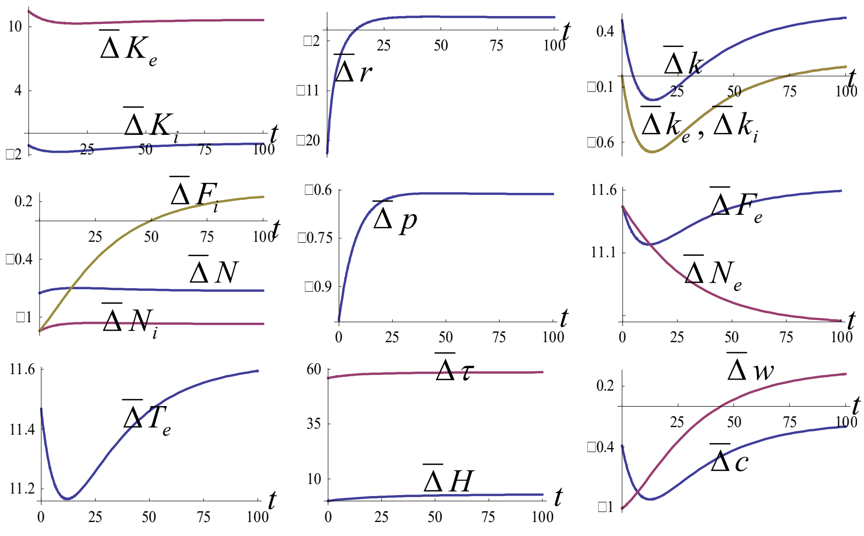


Fig. 4.
A rise in the tax rate on the wage income

A rise in the tax rate on output of the industrial sector

We increase the tax rate on the output level of the industrial sector in the following way: τ_i : 0.005 $\Rightarrow$ 0.015. The simulation results are plotted in Figure 5. By comparing Figures 4 and 5 we notice that in the long term the effects of the change in τ_i and τ_w are the same, except for the education fee. The transitional processes to the equilibrium are different for some variables. For instance, the wage rate and rate of interest initially fall in the case of τ_c , but not in the case of τ_w . It should be remarked that in a study on the interaction between public and private spending in a two-stage education framework (K-12 and tertiary education) with data on education finance in the OECD countries, Arcalean and Schiopu (2010) show that a rise in the overall education public spending crowds out the total level of private contributions. In our dynamic model, we see that the effects are dynamic. As the government increases the tax rate in order to subsidize education, the education fee falls slightly but the education time is much increased. The total private expenditure on education (pT_e) rises over time. The capital input in the education sector rises, but the capital input in the production sector falls. The output level of the production sector falls initially but rises in the long term. It should be noted that in our simulation as education time is increased, the contribution to human capital accumulation due to education is high. Moreover, the return from higher human capital is also large in our case. Our presumed high efficiency in education leads to our positive conclusion about the government's subsidy policy.

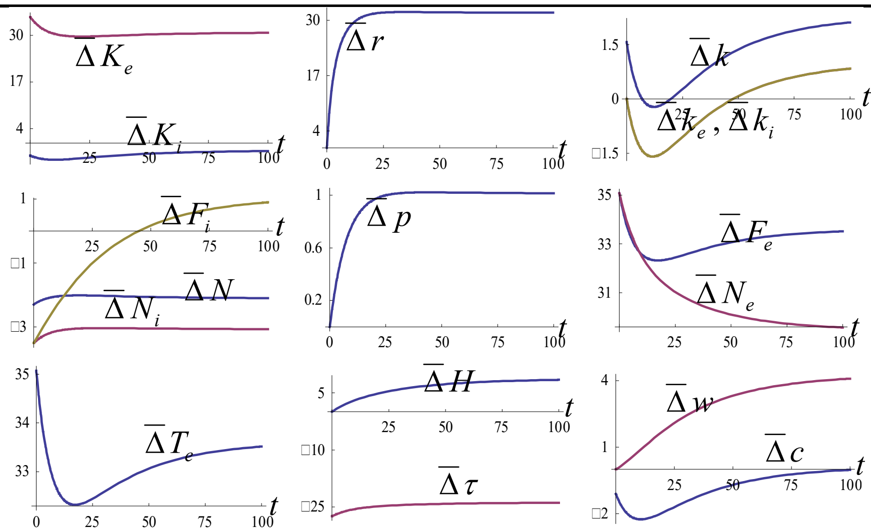
Fig. 5.
A rise in the tax rate on output of the industrial sector



A rise in the propensity to obtain education

It is important to examine effects of changes in the household's preference for education. We allow the propensity to receive education to increase in the following way: $\eta_0 : 0.007 \Rightarrow 0.01$. The simulation results are demonstrated in Figure 6. As people are more interested in obtaining education, they increase education time. As they spend more time and money on education, the education fee is increased and the education sector employs more capital and people and the education sector's output is increased. As people spend more time on formal education, their human capital is increased, which results in rise in wage rate. Although the wage rate, wealth and output levels are all increased (which expands the pie for taxation), the subsidy rate stills falls over time as a consequence of large increases in education time. We see that as the household's propensity to receive education increases, the per capita level of consumption declines first and then increases. This occurs because initially more family resource is devoted to education; but as human capital is increased and wage rate becomes higher, the rise in return from education finally increases consumption. It should be noted that according to Arrow (1973), a stronger interest in education may not lead to human capital and economic growth. The conclusion results from the assumption that students choose education also for the purpose of signaling. In the literature of education and economics, the signaling view of education was initially formally presented by Spence (1973), Arrow (1973), and Stiglitz (1975). This implies that direct productivity gains are not necessary to explain the choice of quantity and quantity of education. A recent research by Lee (2007) shows that signaling may explain why American students study more in college than in high school while the opposite is true for East Asian students. In our human capital accumulation, we assume that a rise in education will make more contribution to human capital accumulation. This explains why our simulation results do not show the phenomenon predicted by Arrow. If we neglect the education part in the capital accumulation, a rise in the propensity to obtain education will lead to social as well as individual wastes. As mentioned by Chanda (2008), over the last three decades returns to higher education have increased while the household savings rate has fallen to almost zero in the US. Chanda builds a representative agent model where savings fall as an outcome of an exogenously driven increase in the return to education. Out model predicts that the total wealth falls only temporarily as people invest more in education. Although the relative fall in the propensity to save tends to reduce the total capital, the return from education implies a rise in the total wealth in the long term.

Fig. 6.
A rise in the propensity to obtain education



Conclusions

This paper built a two-sector growth model with wealth accumulation and human capital accumulation. We emphasize the role of the government's subsidy policies on economic growth and human capital accumulation. This paper made a unique approach to the issue in that it modelled the household's decision on education with an alternative approach proposed by Zhang (1993), and treated the sources of human capital via three ways: Arrow's learning by doing, Uzawa's learning by education, and Zhang's creative leisure within a general equilibrium framework. We studied the general equilibrium among various forces of households' decisions, economic structures, and human capital and wealth changes. Our approach implies that in comparison with most of the theoretical models on the subject with partial analyses our model can properly deal with the impact of government's spending on education in a comprehensive perspective. In our model education is financially supported by public subsidies and the household. Public education's cost is through taxation. The government may tax on households' the private sectors and on wage income, consumption and wealth income. The model describes a dynamic interdependence between wealth accumulation, human capital accumulation, and division of labor under perfect competition. We simulated the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also examined effects of changes in some parameters. We may extend the model in some directions. For instance, we may introduce leisure time as an endogenous variable. Funke and Strulik (2000) propose a formal framework to integrate the two separate lines of research on growth with knowledge – the Uzawa model with education and the endogenous growth models. Influenced by the study by Funke and Strulik, Lacopetta (2010) analyzes the transitional dynamics of a growth model in which both education and innovation are integrated in a single framework. The model identifies various temporal orders of physical capital accumulation, human capital formation, and innovation. Another direction is to introduce heterogeneous households, like Levy (2005) who examines issues related to how society chooses the tax rate and the allocation of the revenues between income redistribution and public education with heterogeneous households (the old and the young).

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Appendix: Proving the Lemma

We now show that the dynamics can be expressed by a two-dimensional differential equations system. From (2) and (5), we obtain

$$\frac{K_e}{N_e} = \alpha \frac{K_i}{N_i}, \ \text{i.e.,} \ k_e = \alpha k_i, \tag{A1}$$

where $\alpha \equiv \alpha_e \beta_i / \alpha_i \beta_e$ ($\neq 1$ assumed). From (A1), (2) and (4), we obtain

$$p(t) = \frac{\bar{\tau}_i \alpha_i A_i}{\bar{\tau}_e \alpha_e A_e} \alpha^{\beta_e} H^{-m\beta} k_i^{\beta}, \tag{A2}$$

where $\beta \equiv \beta_e - \beta_i$. From (A1) and (1), we solve the labor distribution as functions of $k_i(t)$ and $k(t)$

$$n_i = \frac{\alpha k_i - k}{(\alpha - 1)k_i}, \quad n_e = \frac{k - k_i}{(\alpha - 1)k_i}. \tag{A3}$$

Dividing (14) by N_0, we have

$$c + s - \delta \bar{k} = A_i T n_i H^{\beta_i m} k_i^{\alpha_i},$$

where $\delta \equiv 1 - \delta_k$. Substituting $c = \xi \bar{y}$ and $s = \lambda \bar{y}$ into the above equation yields

$$\bar{y} = \left\{ \frac{\alpha A_i H^{\beta_i m} k_i^{\alpha_i}}{\alpha - 1} + \delta k - \frac{A_i H^{\beta_i m} k}{(\alpha - 1)k_i^{\beta_i}} \right\} \frac{T}{\xi + \lambda}, \tag{A4}$$

where we use the equation for n_i in (A3) and $\bar{k} = T k$. Insert (2) and $\bar{k} = T k$ into the definition of $\bar{y}$ in (8)

$$\bar{y} = \left(1 - \bar{\tau}_h \delta_k + \bar{\tau}_h \bar{\tau}_i \alpha_i A_i H^{m\beta_i} k_i^{-\beta_i}\right) k T + \bar{\tau}_w \bar{\tau}_i T_0 \beta_i A_i H^{m\beta_i} k_i^{\alpha_i}. \tag{A5}$$

From (A4) and (A5), we solve

$$\left(\alpha A_\xi H^{\beta_i m} k_i^{\alpha_i} + \bar{\delta} k - \frac{A_\xi H^{\beta_i m} k}{k_i^{\beta_i}} - \bar{\tau}_h \bar{\tau}_i \alpha_i A_i H^{m\beta_i} k_i^{-\beta_i} k \right) T \tag{A6}$$
$$= \bar{\tau}_w \bar{\tau}_i T_0 \beta_i A_i H^{m\beta_i} k_i^{\alpha_i},$$

where

$$\bar{\delta} \equiv \frac{\delta}{\xi + \lambda} - \left(1 - \bar{\tau}_h \delta_k\right), \quad A_\xi \equiv \frac{A_i}{(\alpha - 1)(\xi + \lambda)}.$$

From (12) and (4), we have

$$T_e = A_e \, T \, n_e \, H^{\beta_e m} \, k_e^{\alpha_e}. \qquad \textbf{(A7)}$$

Insert $T + T_e = T_0$ and n_e in (A3) in (A7)

$$T = \left(1 + \frac{\alpha^{\alpha_e} \, A_e \, H^{\beta_e m} (k - k_i)}{(\alpha - 1) k_i^{\beta_e}} \right)^{-1} T_0. \qquad \textbf{(A8)}$$

Substituting (A8) into (A6) yields

$$k = \varphi(k_i, H) \equiv \frac{\left(1 - \alpha^{\alpha_e} A_e \, k_i^{\alpha_e} \, H^{\beta_e m} / (\alpha - 1)\right) \bar{\tau}_w \, \bar{\tau}_i \, \beta_i A_i \, k_i \; - \alpha \, A_\xi \, k_i}{\bar{\delta} \, k_i^{\beta_i} \, H^{-\beta_i m} - A_\xi - \bar{\tau}_h \, \bar{\tau}_i \, \alpha_i A_i - \alpha^{\alpha_e} \, \bar{\tau}_w \, \bar{\tau}_i \, \beta_i A_i \, A_e \, k_i^{\alpha_e} \, H^{\beta_e m} / (\alpha - 1)}. \qquad \textbf{(A9)}$$

By (A9), we can express $k(t)$ as functions of $k_i(t)$ and $H(t)$ at any point of time. By (A8) and (A5), we can also express $T(t)$ and $\bar{y}(t)$ as functions of $k_i(t)$ and $H(t)$ as follows

$$T = \varphi_0(k_i, H), \;\; \bar{y} = \Lambda(k_i, H). \qquad \textbf{(A10)}$$

These functions show that $T(t)$, $\bar{y}(t)$, $N(t) (= T(t) N_0)$, and $K(t)$ $(= k(t) N(t))$ can be treated as functions of $k_i(t)$ and $H(t)$ at any point of time. By (A3) and $N_j(t) = n_j(t) N(t)$, we see that the labor distribution, $n_j(t)$ and $N_j(t)$ $(j = i, s)$, are functions of $k_i(t)$ and $H(t)$. It is straightforward to see that $F_j(t)$ and $C(t)$ can be expressed as functions of $k_i(t)$ and $H(t)$ at any point of time. Also with (3), it is straightforward to show that the motion of human capital can be expressed as a function of $k_i(t)$ and $H(t)$ at any point of time

$$\dot{H}(t) = \tilde{\Omega}_h(k_i(t), H(t)).$$

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We now show that change in $k_i(t)$ can also be expressed as a differential equation in terms of $k_i(t)$ and $H(t)$. First, substitute $\bar{y} = \Lambda$ and $\bar{k} = Tk = \varphi_0 \varphi$ into (11)

$$\dot{\bar{k}}(t) = \lambda \Lambda(k_i(t), H(t)) - \varphi_0(k_i(t), H(t))\varphi(k_i(t), H(t)). \qquad \textbf{(A11)}$$

Taking derivatives of $\bar{k} = T k = \varphi_0 \varphi$ with respect to time, we have

$$\dot{\bar{k}} = \left(\frac{\partial \varphi_0}{\partial k_i}\varphi + \frac{\partial \varphi}{\partial k_i}\varphi_0\right)\dot{k}_i + \left(\frac{\partial \varphi_0}{\partial H}\varphi + \frac{\partial \varphi}{\partial H}\varphi_0\right)\tilde{\Omega}_h, \qquad \textbf{(A12)}$$

where we use (A10). Substituting (A12) into (A11) yields

$$\dot{k}_i = \tilde{\Omega}_i(k_i, H) \equiv$$
$$\left[\lambda \Lambda - \varphi_0 \varphi - \left(\frac{\partial \varphi_0}{\partial H}\varphi + \frac{\partial \varphi}{\partial H}\varphi_0\right)\tilde{\Omega}_h\right]\left(\frac{\partial \varphi_0}{\partial k_i}\varphi + \frac{\partial \varphi}{\partial k_i}\varphi_0\right)^{-1}. \qquad \textbf{(A13)}$$

The two differential equations (A10) and (A13) contain two variables $k_i(t)$ and $H(t)$. We thus proved Lemma 1.